

## Charmed Hadron Interactions

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We compute the scattering lengths of charmed mesons and charmonia scattering with light hadrons in full QCD. We use Fermilab formulation for the charm quark and domain-wall fermions for the light quarks and staggered sea quarks. Four different light-quark masses are used to extrapolate to the physical point. The charmed baryon spectrum is also presented.

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## 1. Introduction

Lattice QCD calculations of the properties of hadronic interactions such as elastic scattering phases shifts and scattering lengths have recently started to develop. Precision results have been obtained in the meson-meson sector for certain processes such as pion-pion, kaon-kaon and pion-kaon scattering and preliminary results for baryon-baryon scattering lengths have been presented. A recent review of these calculations can be found in [1]. In this work we study scattering processes where one hadron contain a charm quark. Firstly, we study the scattering processes of charmonia ( $\eta_c$  and  $J/\Psi$ ) with light hadrons ( $\pi$ ,  $\rho$  and  $N$ ). As it has been pointed out in the literature [2, 3, 4], such interaction has a direct relation to possible charmonium-nucleus bound states with binding energy of a few MeV. Unlike the traditional nuclear force that binds nucleons, in this case, there are no quark exchange diagrams, and only gluons are responsible for the binding. In other words, the charmonium nucleon force is purely a gluonic van der Waals force. The charmonium interactions with light hadrons has been also studied in quenched lattice QCD [5]. Secondly, we study the scattering processes of charmed mesons ( $D_s$  and  $D$ ) with light mesons ( $\pi$  and  $K$ ). In reference [6],  $D\pi$  and  $DK$  scattering were studied using the scalar form factors in semileptonic pseudoscalar-to-pseudoscalar decays. In our work we use Lüscher's formula [7] to extract scattering information from the energy shift of two interacting hadrons relative to the total energy of the two individual hadrons.

## 2. Fermion Actions

We use Fermilab formulation [8] for the charm quark, domain-wall fermions for the light quarks and staggered sea quarks. In the following, we will specify the Fermilab formulation as well as the tuning of the parameters in this formulation. The action is:

$$\begin{aligned}
 S &= S_0 + S_B + S_E, \\
 S_0 &= \sum_x \bar{q}(x) [m_0 + (\gamma_0 \nabla_0 - \frac{b}{2} \Delta_0) + v \sum_i (\gamma_i \nabla_i - \frac{b}{2} \Delta_i)] q(x), \\
 S_B &= -\frac{b}{2} c_B \sum_x \bar{q}(x) (\sum_{i<j} \sigma_{ij} F_{ij}) q(x), \\
 S_E &= -\frac{b}{2} c_E \sum_x \bar{q}(x) (\sum_i \sigma_{0i} F_{0i}) q(x),
 \end{aligned}$$

where  $b$  is the lattice spacing,  $\nabla_0$  and  $\nabla_i$  are first-order lattice derivatives in time direction and space directions,  $\Delta_0$  and  $\Delta_i$  are second-order lattice derivatives,  $F_{\mu\nu}$  is the field tensor defined in reference [8]. In this action, the space-time exchange symmetry is not imposed.  $S_0$  is just the standard Wilson fermion action except that the coefficient in front of the space term and the coefficient in front of the temporal term are different.  $S_B$  and  $S_E$  are spatial and temporal clover terms, also with different coefficients.

There are four parameters to tune: the charm quark mass  $m_c$ , the anisotropy  $v$ , and the two clover coefficients  $c_B$  and  $c_E$ . We use the spin-average mass of  $J/\Psi$  and  $\eta_c$  to tune the charm quark mass. The value of  $v$  is tuned to restore the dispersion relation. As for  $c_B$  and  $c_E$ , the tree level

tadpole improvement estimate is  $c_B = c_E = 1/u_0^3$ . Chen suggested a better way to evaluate the two parameters [9]:

$$c_B = \frac{v}{u_0^3}, \quad c_E = \frac{1}{2}(1+v)\frac{1}{u_0^3}. \quad (2.1)$$

Here  $c_B$  and  $c_E$  both depend on  $v$ . We use Chen's evaluation in our work.

### 3. Numerical Ensembles

We employ the gauge configurations generated by the MILC Collaboration [10]. We use the  $20^3 \times 64$  lattices generated at four values of light-quark masses. The lattice spacing  $b = 0.12406$  fm. The details of the ensembles are listed below:

Ensemble	$bm_l$	$bm_s$	$bm_l^{dwf}$	$bm_s^{dwf}$	number of props
2064f21b676m007m050	0.007	0.050	0.0081	0.081	450
2064f21b676m010m050	0.010	0.050	0.0138	0.081	650
2064f21b679m020m050	0.020	0.050	0.0313	0.081	550
2064f21b781m030m050	0.030	0.050	0.0478	0.081	380

The subscript  $l$  denotes light quark, and  $s$  denotes the strange quark. The superscript  $dwf$  denotes domain-wall fermion.

### 4. Heavy-Quark Action Test

In the heavy-quark action, the dispersion relation  $E^2 = m^2 + c^2 p^2$  needs to be restored by tuning the parameter  $v$  to get the value of  $c^2$  to agree with the theoretical value 1. To do that, we calculate the single-particle energy of  $\eta_c, J/\Psi, D_s$  and  $D$  at the six lowest momenta:  $\frac{2\pi}{L}(0,0,0), \frac{2\pi}{L}(1,0,0), \frac{2\pi}{L}(1,1,0), \frac{2\pi}{L}(1,1,1), \frac{2\pi}{L}(2,0,0), \frac{2\pi}{L}(2,1,0)$ . We tune the dispersion relation of  $\eta_c$  and get the dispersion relations of  $J/\Psi, D_s$  and  $D$  to be restored as well. The following table lists the values of  $c^2$  we get from the numerical simulation:

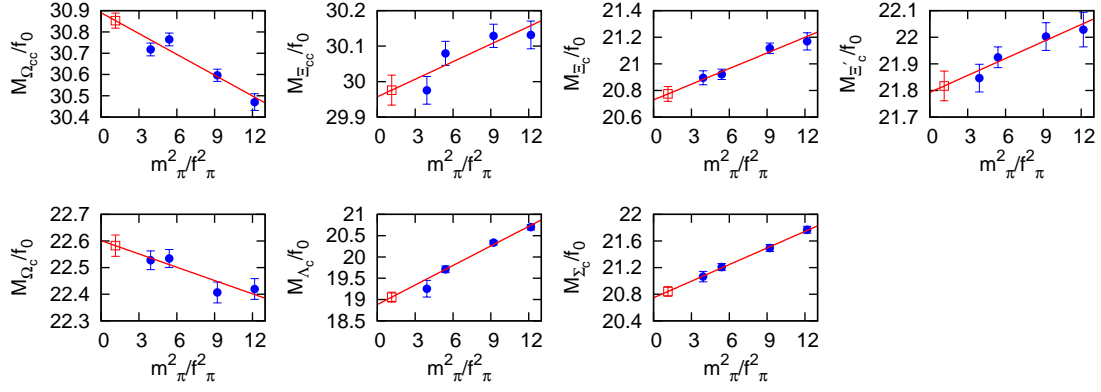
	$\eta_c$	$J/\Psi$	$D$	$D_s$
$c^2$	0.989(0.005)	0.965(0.009)	1.012(0.017)	1.006(0.009)

By tuning the charm-quark mass, we get the spin-average mass of  $\eta_c$  and  $J/\Psi$  to be 3056.54(1.15)MeV, which agree well with the experimental value. We also calculate the hyperfine splitting, which we find to be 99.1(1.1)MeV.

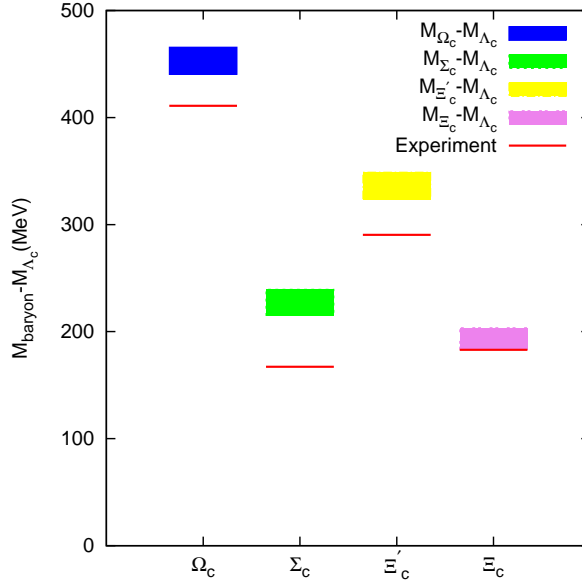
In this section, all numerical results are obtained on the ensemble 2064f21b676m007m050 which has the lightest light-quark mass. We keep the charm quark mass and anisotropy parameter fixed for all ensembles.

### 5. Baryon Spectrum

We calculate the masses of singly charmed baryons and doubly charmed baryons at four different light quark masses. We employ a simple linear relation  $m_{baryon} = c_1 + c_2 m_\pi^2 / f_\pi^2$  to extrapolate the baryon masses to the physical point. The values of  $m_\pi / f_\pi$  are taken from reference [11]. Fig. 1 shows the baryon mass spectrum. We present the singly charmed baryon mass splittings in Fig. 2. Our results are comparable with similar work done using a staggered light-quark action [12].



**Figure 1:** Charmed baryon masses as functions of  $m_\pi^2/f_\pi^2$ . The masses are divided by  $f_0 = 130.7\text{MeV}$  to make them dimensionless. The four blue points in each panel denote the baryon masses at four different light-quark masses. They are extrapolated to the physical values, which are denoted by the red points.



**Figure 2:** Mass splittings of singly charmed baryons. The red lines represent the experimental values. The four bars in different colors represent the numerical values of the mass difference between  $\Lambda_c$  and other singly charmed baryons. The heights of the bars indicate the statistical errors.

## 6. Charmed Hadron Interactions

Lüscher has shown that the scattering phase shift is related to the energy shift ( $\Delta E$ ) of two interacting hadrons relative to the total energy of the two individual hadrons [7]. The total energy of two interacting hadrons ( $h_1$  and  $h_2$ ) is obtained from the four-point correlation function:

$$G^{h_1-h_2}(t) = \langle \mathcal{O}^{h_1}(t) \mathcal{O}^{h_2}(t) (\mathcal{O}^{h_1}(0) \mathcal{O}^{h_2}(0))^\dagger \rangle. \quad (6.1)$$

To extract the energy shift  $\Delta E$ , we define a ratio  $R^{h_1-h_2}(t)$ :

$$R^{h_1-h_2}(t) = \frac{G^{h_1-h_2}(t,0)}{G^{h_1}(t,0)G^{h_2}(t,0)} \longrightarrow \exp(-\Delta E \cdot t), \quad (6.2)$$

where  $G^{h_1}(t,0)$  and  $G^{h_2}(t,0)$  are two-point functions.  $\Delta E$  can be obtained by fitting  $R^{h_1-h_2}(t)$  to a single exponential.

The magnitude of center-of-mass momentum  $p$  is related to  $\Delta E$  by

$$\Delta E = \sqrt{p^2 + m_{h_1}^2} + \sqrt{p^2 + m_{h_2}^2} - m_{h_1} - m_{h_2}. \quad (6.3)$$

The phase shift is obtained from the following relation:

$$p \cot \delta(p) = \frac{1}{\pi L} \mathbf{S} \left( \left( \frac{pL}{2\pi} \right)^2 \right), \quad (6.4)$$

where the  $\mathbf{S}$  function is defined as

$$\mathbf{S}(x) = \sum_{|\mathbf{j}| < \Lambda} \frac{1}{|\mathbf{j}|^2 - x} - 4\pi\Lambda. \quad (6.5)$$

The sum is over all three-vectors of integers  $\mathbf{j}$  such that  $|\mathbf{j}| < \Lambda$ , and the limit  $\Lambda \rightarrow \infty$  is implicit. If the interaction range is smaller than half of the lattice size, the  $s$ -wave phase shift will be

$$p \cot \delta(p) = \frac{1}{a} + \mathcal{O}(p^2), \quad (6.6)$$

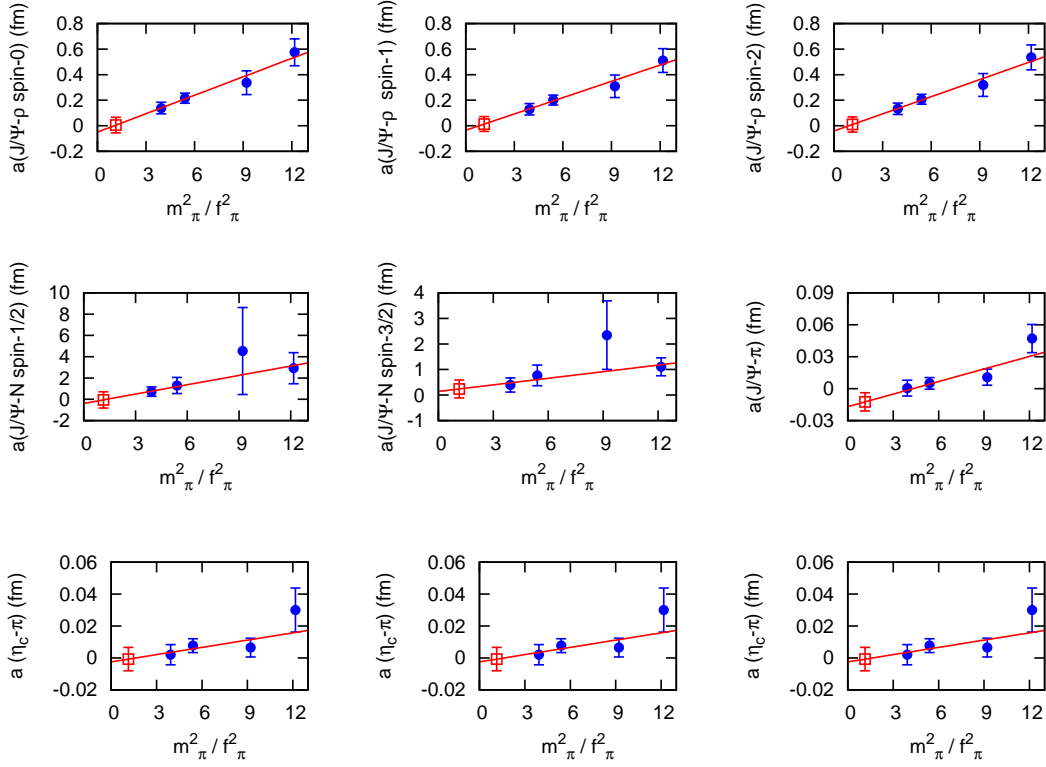
where  $a$  is the scattering length.

By measuring the energy shift, the momentum  $p$  can be obtained by equation (6.3). Then we calculate the right-hand side of equation (6.4). From equation (6.4) and equation (6.6), we can see that the scattering length is just the inverse of the right-hand side of equation (6.4).

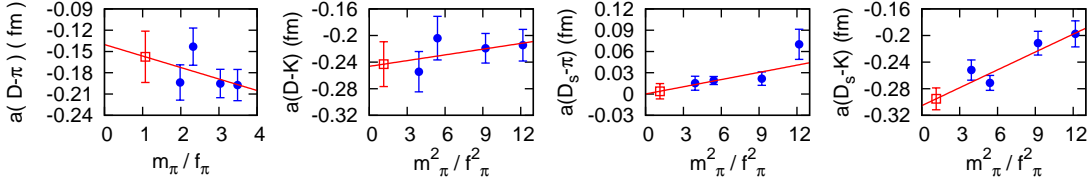
We study the scattering of the charmonia ( $\eta_c, J/\Psi$ ) with light hadrons ( $\pi, \rho, N$ ). The scattering of charmed mesons ( $D_s, D$ ) with  $\pi$  and  $K$  are also studied. We choose the isospin- $\frac{3}{2}$  channel for  $D - \pi$  scattering and isospin-1 channel for  $D - K$  scattering to avoid disconnected diagrams. For the same reason, we use  $D_s^+$  and  $K^+$  for  $D_s - K$  scattering. The interactions of the heavy hadron ( $h$ ) with the pion has a special feature due to the Nambu-Goldstone nature of the pion. The  $s$ -wave scattering length is given by [13]

$$a^{h-\pi} = -\left(1 + \frac{m_\pi}{m_h}\right)^{-1} \frac{m_\pi}{8\pi f_\pi^2} [I(I+1) - I_h(I_h+1) - 2] + \mathcal{O}(m_\pi^2), \quad (6.7)$$

where  $I$  is the total isospin of the  $\pi - h$  system. The first term in equation (6.7) vanishes for  $J/\Psi - \pi$ ,  $\eta_c - \pi$  and  $D_s - \pi$  channels but not for  $D - \pi$  channel. So the leading term of  $D - \pi$  scattering length is proportional to  $m_\pi/f_\pi$  while the leading terms of  $J/\Psi - \pi$ ,  $\eta_c - \pi$  and  $D_s - \pi$  scattering lengths are proportional to  $m_\pi^2/f_\pi^2$ . We measure the scattering lengths at four different light-quark masses. We use the relation  $a = c_1 + c_2 m_\pi/f_\pi$  to extrapolate  $D - \pi$  scattering length to the physical point. For other channels, we use the relation  $a = c_1 + c_2 m_\pi^2/f_\pi^2$  to do the extrapolations because the leading term in equation (6.7) vanishes. Fig. 3 and Fig. 4 show the scattering lengths of all these channels.



**Figure 3:** Charmonium to pion, rho and nucleon scattering lengths as functions of  $m_\pi^2/f_\pi^2$ . In each panel, the four blue points denote the scattering length measured at four different light-quark masses. The red points denote the extrapolated values.



**Figure 4:** Charmed meson scattering lengths as functions of  $m_\pi^2/f_\pi^2 (m_\pi/f_\pi)$ . In each panel, the four blue points denote the scattering length measured at four different light-quark masses. The red points denote the extrapolated values.

The following tables list the extrapolated scattering lengths of all channels:

Channel	Scattering lengths(fm)	Channel	Scattering lengths(fm)
$\eta_c - \pi$	0.00(1)	$J/\Psi - \rho$ spin 1	0.01(6)
$\eta_c - \rho$	0.03(5)	$J/\Psi - \rho$ spin 2	0.01(6)
$\eta_c - N$	0.18(9)	$D - \pi$	-0.16(4)
$J/\Psi - \pi$	-0.01(1)	$D - K$	-0.23(4)
$J/\Psi - N$ spin 1/2	-0.05(77)	$D_s - \pi$	0.00(1)
$J/\Psi - N$ spin 3/2	0.24(35)	$D_s - K$	-0.31(2)
$J/\Psi - \rho$ spin 0	0.00(6)		

## 7. Conclusion

We calculated the scattering lengths of charmed mesons and charmonia with light hadrons. For the channels of charmonia with light hadrons and  $D_s - \pi$  channel, we found weak interactions. The scattering lengths are zero or close to zero. For the  $D - \pi$ ,  $D - K$  and  $D_s - K$  channels, we found relatively strong repulsive interactions. In the future, we will improve statistics to get more reliable results, and perform extrapolations using chiral perturbation theory forms.

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